# Prediction using Kalman Filter

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**Abstract**

Kalman filtering was developed in the 1960s, although it has its roots as far back as Karl Gauss in 1795.Kalman filtering has been applied in areas as diverse as aerospace, marine navigation, nuclear power plant instrumentation, demographic modeling, manufacturing, and many others.The Kalman filter is the best linear filter given the mean and standard deviation of the noise.In this paper,the Kalman Filter will be used to predict the closing price of an exchange traded fund. (NYSE symbol:FAS)

## 1 Introduction

Filtering theory was initially used to filter noises from radio communications signals. A good filtering algorithm can remove the noise from electromagnetic signals while still retaining useful information. Recently it has been used to predict the future price of stocks (time series) under the assumption that it is the output of a dynamic system with noise. The Kalman filter is an algorithm for efficiently doing exact inference in a linear dynamic system, which is similar to a hidden Markov model but where the state space of the latent variables is discrete and where all latent and observed variables have a Gaussian distribution. So, with the assumption that the time series is linear and Gaussian, we have applied the Kalman Filter to predict the closing price of an exchange traded fund. (NYSE symbol:FAS)

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The Kalman Filter is a two-stage algorithm that assumes there is a smooth trend-line within the data that represents the true value of the stock before being disturbed by noise. In the first stage, a few previous trend-line values are fit to a model. This is then extrapolated to the next time value to generate a prediction and its error variance. In the next stage, the corresponding data value is read and a new trend value is computed as a weighted average of the prediction and actual data value. The weightage is based on the relative amounts of noise in the data and predictions. The filter then repeats this cycle of prediction and correction as each new data value is observed.

## 2 Dynamic System Model

Kalman filters are based on discrete linear dynamic systems. They are often modeled on a Markov chain with linear operators and Gaussian noise. At each time increment(discrete), this linear operator is applied to the current state to get the new state with some noise. As mentioned earlier,the Kalman filter is similar to a Hidden Markov model(HMM) with one main difference. A Hidden Markov model can represent any arbitrary distribution for the next value whereas in the Kalman filter it is Gaussian.
The Kalman filter assumes that the state at time $k$ evolves from the state at time *(k-1)* according to

$$X_k = F_k X_{k-1} + w_k$$

where

- $\mathbf{F}_k$ is the state transition model
- $\mathbf{w}_k$ is the process noise with $w_k \sim \mathcal{N}(0, Q_k)$

This equation is called the dynamic (plant) equation or the system equation.

At time $k$ an observation $Y_k$ is made which follows

$$Y_k = H_k X_k + v_k$$

where

- $H_k$ is the observation model
- $v_k$ is the observation noise with $v_k \sim \mathcal{N}(0, R_k)$

This is the observation or the measurement equation.

The initial state $X_0$ is generally modelled as a random variable, Gaussian distributed with known mean and covariance.The two noise sequences and the initial state are mutually independent. This is the **linear Gaussian assumption**. The matrices $F_k$ and $H_k$ are system and measurement dependent respectively.

# 3 Kalman Filter

## 3.1 The Algorithm

The Kalman filter uses the previous estimated state and the current measurement to calculate the estimate of the current state. This means that it doesn't need historical values like a batch estimator and hence it is a recursive estimator. The state is represented by two variables: $X_k$, which is the state estimate given observations till time $k$ and $P_k$, which is the estimated error covariance matrix or in simple words the a measure of the estimated accuracy of the state estimate. There are two stages in the Kalman filter algorithm namely:

1. **Prediction**

2. **Correction**

### 3.1.1 Prediction

**1.** The predicted state estimate is given by $\hat{X}_k = F_k X_{k-1}$

**2.** The predicted covariance estimate is given by $\hat{P}_k = F_k P_{k-1} F_k^T$

The variables $\hat{X}$ and $\hat{P}$ denote the predicted values and the variables $X$ and $P$ denote the corrected values from the previous iteration (or the initial values when *k=1*)

### 3.1.2 Correction

**1.** The measurement residual is given by $V_k = Y_k - H_k \hat{X}_k$

**2.** The residual covariance is given by $S_k = H_k \hat{P}_k H_k^T + R_k$

**3.** The Kalman Gain is given by $W_k = \hat{P}_k H_k^T S_k^{-1}$

**4.** The corrected state estimate is computed as $X_k = \hat{X}_k + W_k V_k$

**5.** The corrected covariance estimate is computed as $P_k = \hat{P}_k - W_k S_k W_k^T$

Here $Y_k$ is the measurement(observed value of the stock) and the difference between the measurement and the predicted state estimate is the measurement residual $V_k$. The noises $Q_k$ and $R_k$ are taken such that the variance of the series $(Y_k - \hat{X}_k)$ is minimized. The matrices $F_k$ and $H_k$ are chosen according to the historical prices of the stock. Also since the values of $Q_k, R_k, F_k, H_k$ are fixed throughout the recursion, the time indices $(k)$ can be dropped.

### 3.2 Kalman Gain

The optimal filter gain is :

- High, if the state prediction has large variance and the measurement has relatively small variance.
- Low, if the state prediction has small variance and the measurement has relatively large variance.

Simply put, when the filter gain is High, there is a very quick response to the measurement while updating the state and when the filter gain is Low, there is a slower response to the measurement. In other words, a model that fits the data well will lead to a better prediction, Low filter gain and hence better noise reduction. Whereas a filter with a very high gain relies more on the measurement and hence yields less noise reduction. The Kalman gain found in the algorithm is the *Optimal* Kalman gain. If any other gain is used, the equations given here for corrected state and covariance estimate cannot be used.

## 4 Results

The closing prices for 150 days was predicted using the Kalman filter described above.The results obtained are below and self explanatory.

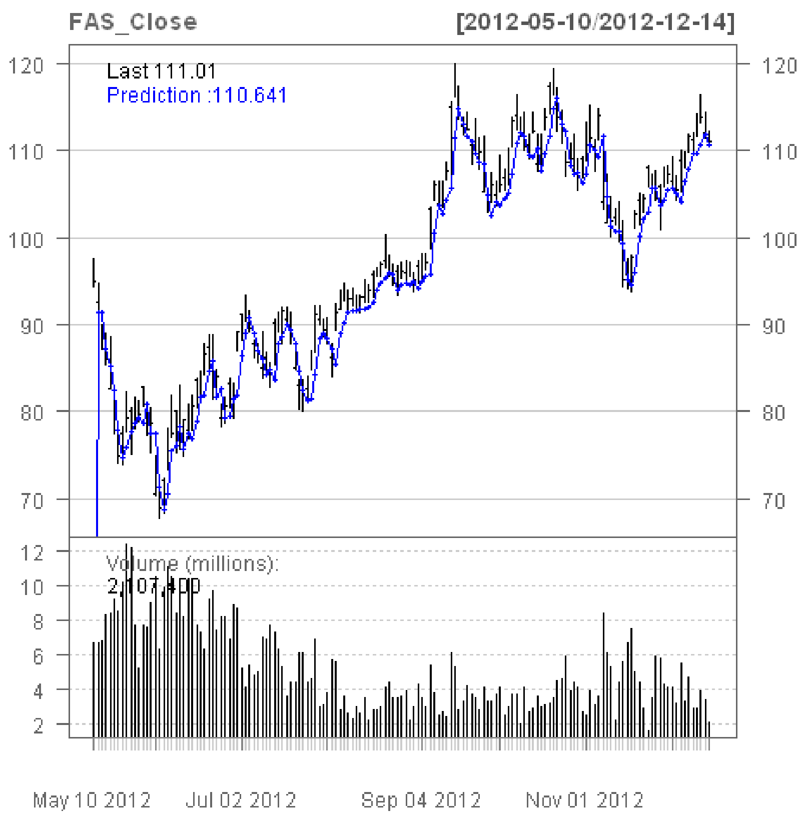


Figure 1: Predicted vs Actual Closing prices of FAS.

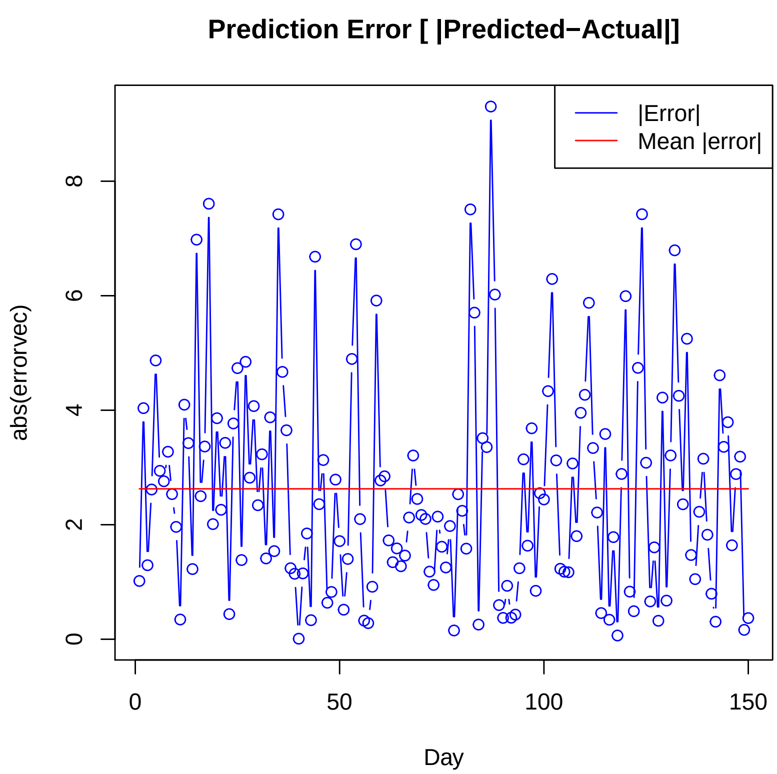


Figure 2: Prediction Error and mean of the prediction error

## 5 Conclusion and Future Work

It is clear that Kalman filter gives very good predictions for the price of stock at *t+1*. To use it for *t+2, t+3, t+4, etc* would require a lot of assumptions which will eventually lead to bad predictions. In this model of Kalman filter we have just used one lag, *i.e* I have assumed that the future value depends only on the current value. Future work can be done by assuming future value depends on some of the past values (*t-1,t-2,t-3 etc*). Also, the Kalman filter assumes linearity between the data and Gaussian noise which are not very good assumptions. In order to use realistic assumptions about the price dynamics,a particle filter can be implemented for the same process of predicting stocks as the particle filter doesn't assume linearity or the noise being Gaussian.

## 6 References


1. Estimation with Applications to Tracking and Navigation - Yaakov Bar-Shalmom, X.Rong Li and Thiagalingam Kirubarajan.

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